

Target Tracking

Le 4: Multi-Target Tracking: single-hypothesis tracking

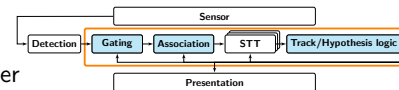
Gustaf Hendeby

Div. Automatic Control
Dept. Electrical Engineering
gustaf.hendeby@liu.se

- 1 Multi-Target Tracking
- 2 Global Nearest Neighbor
- 3 Joint Probabilistic Data Association
- 4 Exercises

Summary: lecture 3

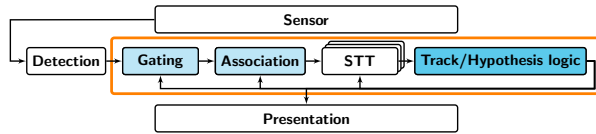
- **Gate** to improve complexity in presence of clutter
 - Rectangular: cheap but crude
 - Ellipsoidal: more correct
- **Track logic** determines if there is an object present or not
 - State-machine for confirming target, based on gated measurements
 - Score based logic, based on a hypothesis test
- Different **association** strategies exist (so far for **STT**)
 - Nearest neighbor (NN) association
A hard decision to use the "closest" measurement.
 - Probabilistic data association (PDA)
A soft decision where all measurements in the gate are combined.



References on Multiple Target Tracking Topics

- D. Bertsekas. **Auction algorithms**. URL http://www.mit.edu/~dimitrib/Auction_Encycl.pdf (Auction algorithm)
- B.-N. Vo, M. Mallick, Y. Bar-Shalom, S. Coraluppi, R. Osborne, III, R. Mahler, and B.-T. Vo. **Multitarget Tracking**. Wiley Encyclopedia of Electrical and Electronics Engineering, 2015. URL https://www.researchgate.net/publication/283623828_Multitarget_Tracking (MTT, GNN)
- Y. Bar-Shalom, F. Daum, and J. Huang. **The probabilistic data association filter**. *IEEE Control Systems Magazine*, 29(6):82–100, Nov. 2009. (PDA/JPDA)

Multi-Target Tracking



Association: a multi target tracking perspective

Definition: association

Association is the process of assigning measurements to existing tracks or existing tracks to measurements (measurement-to-track association vs. track-to-measurement association).

- In the classical air traffic control (ATC) application, there are hundreds of targets and measurements.
- The number of possible combinations of measurements and targets grows combinatorally.
- Not all associations are likely or even feasible.
- Very unlikely combinations should be removed as soon as possible!

Association Hypothesis

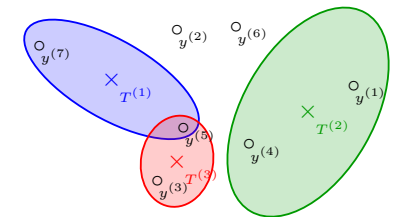
Definition: association hypothesis

An (association) **hypothesis** is a partitioning of a set of measurements according to their origin; individual existing targets, clutter/false detections, and new targets.

- A **single hypothesis tracker** (SHT) maintains a *single* hypothesis about all of the measurements received over time.
 - The **global nearest neighbor** (GNN) algorithm does this by selecting the best hypothesis according to a criterion.
 - The **joint probabilistic data association** (JPDA) filter combines all possible current hypotheses into a single hypothesis.
- A **multiple hypothesis tracker** (MHT), maintains *multiple* hypotheses about the origin of the received measurements.

Multi-Target Association: example

- Using STT for each target, results in locally optimal solutions, which might be infeasible. Consider the association hypothesis: $T^{(1)} \leftrightarrow y^{(5)}$, $T^{(2)} \leftrightarrow y^{(1)}$, $T^{(3)} \leftrightarrow y^{(5)}$ which picks the best measurement for each target, but violates the assumption that a measurement originates from a single target.
- In MTT the complete association hypothesis is considered, to obtain a global optimum and avoid infeasible solutions.



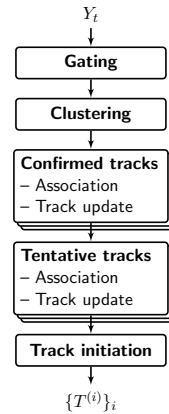
Example with three targets, $T^{(1)}, \dots, T^{(3)}$, and seven measurements $y^{(1)}, \dots, y^{(7)}$.

Track logic and gating will be utilized to simplify the MTT process.

Single Hypothesis Tracking

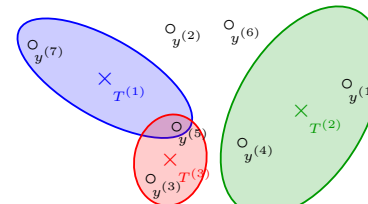
Principal steps:

- Gating**
Gating is performed, yielding a validation matrix \mathcal{V} indicating with measurements should be considered for each track.
- Clustering**
Tracks that do not share potential measurements are separated, yielding many smaller problems.
- Association and updating of confirmed tracks**
Associate measurements to confirmed tracks and update the tracks. From now on, do not consider any measurements that has been gated with a confirmed track.
- Association and updating of tentative tracks**
Update the procedure with the remaining measurements and the tentative tracks.
- Initiate new tentative tracks**
Use remaining measurements to start tentative tracks.



Gating and Validation Matrix

- Perform gating between all measurements and targets (using suitable gating strategy).
- Create the validation matrix \mathcal{V} , where each element indicate if the measurement and track are compatible or not.
- The validation matrix is used to create the assignment hypothesis.



	$T^{(1)}$	$T^{(2)}$	$T^{(3)}$
$y^{(1)}$	0	1	0
$y^{(2)}$	0	0	0
$y^{(3)}$	0	0	1
$y^{(4)}$	0	1	0
$y^{(5)}$	1	0	1
$y^{(6)}$	0	0	0
$y^{(7)}$	1	0	0

- Validation matrix, \mathcal{V}

Example of gating and resulting validation matrix

Assignment: notation

Measurement origins

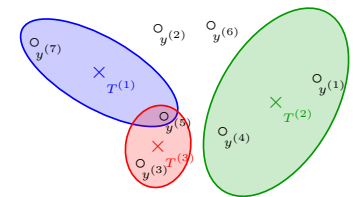
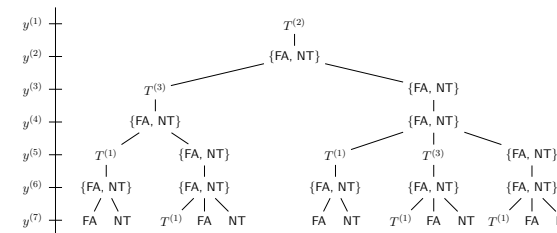
If we consider measurements in a scan and existing tracks:

- TC** Track Continuation: a measurement will update a track
- FA** False Alarm: a measurement is considered as nuisance
- NT** New Track: a measurement can start a new track

It is reasonable to assume that a measurement can only be used for one of the above.

Possible Association Hypotheses

Ex: consider the case where $y_t^{(1)}$ is associated to $T^{(2)}$.



	$T^{(1)}$	$T^{(2)}$	$T^{(3)}$
$y^{(1)}$	0	1	0
$y^{(2)}$	0	0	0
$y^{(3)}$	0	0	1
$y^{(4)}$	0	1	0
$y^{(5)}$	1	0	1
$y^{(6)}$	0	0	0
$y^{(7)}$	1	0	0

Validation matrix, \mathcal{V}

To complete: repeat for $y_t^{(1)} = FA$ and $y_t^{(1)} = NT$, and $\{FA, NT\}$ indicates that FA and NT yields identical subtrees.

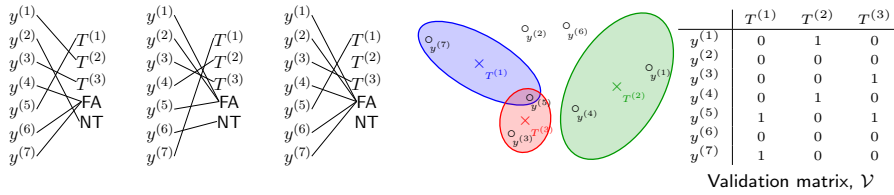
Association Hypothesis: example

Define the **association hypothesis** θ_t as a mapping

$$\theta_t(\cdot) : \{1, 2, \dots, m_t\} \rightarrow \{\text{FA}, 1, 2, \dots, n_t, \text{NT}\}$$

- m_t is the number of measurements in (scan) Y_t , i.e., $Y_t = \{y_t^{(1)}, \dots, y_t^{(m_t)}\}$
- n_t is the number of tracks when entering the frame.

Example: hypotheses when $m_t = 7, n_t = 3$

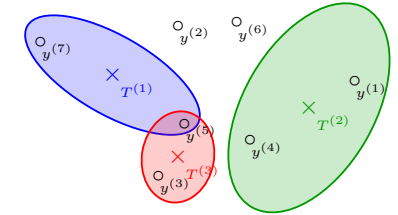


Clustering

- Computational complexity scales exponentially with number of measurements and targets.
- Tracks that do not share any measurements can be treated separately, to reduce the complexity.
- Clusters in the example: $\mathcal{C}^{(1)} = \{T^{(1)}, T^{(3)}\}$, $\mathcal{C}^{(2)} = \{T^{(2)}\}$.

	T_1	$T^{(2)}$	T_3
$y^{(1)}$	0	1	0
$y^{(2)}$	0	0	0
$y^{(3)}$	0	0	1
$y^{(4)}$	0	1	0
$y^{(5)}$	1	0	1
$y^{(6)}$	0	0	0
$y^{(7)}$	1	0	0

Validation matrix, \mathcal{V}



$y^{(2)}$ and $y^{(6)}$ do not fit any gate and can only be FA or NT.

Clustering

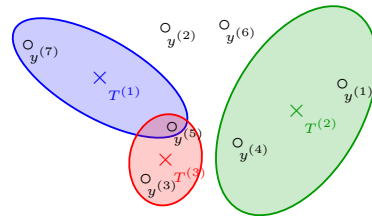
- Computational complexity scales exponentially with number of measurements and targets.
- Tracks that do not share any measurements can be treated separately, to reduce the complexity.
- Clusters in the example: $\mathcal{C}^{(1)} = \{T^{(1)}, T^{(3)}\}$, $\mathcal{C}^{(2)} = \{T^{(2)}\}$.

	T_1	T_3
$y^{(3)}$	0	1
$y^{(5)}$	1	1
$y^{(7)}$	1	0

Validation matrix, $\mathcal{V}^{(1)}$

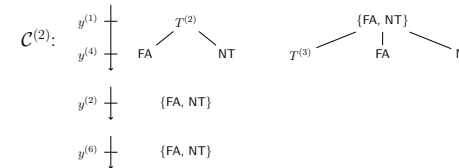
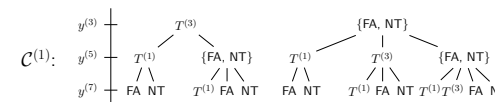
	$T^{(2)}$
$y^{(1)}$	1
$y^{(4)}$	1

Validation matrix, $\mathcal{V}^{(2)}$



$y^{(2)}$ and $y^{(6)}$ do not fit any gate and can only be FA or NT.

Association Hypotheses: revisited using clustering



	T_1	T_3
$y^{(3)}$	0	1
$y^{(5)}$	1	1
$y^{(7)}$	1	0

Validation matrix, $\mathcal{V}^{(1)}$

	$T^{(2)}$
$y^{(1)}$	1
$y^{(4)}$	1

Validation matrix, $\mathcal{V}^{(2)}$

The selections in the respective clusters can be made independently!

Hypothesis Probabilities: track continuation

Track Continuation (TC)

- Detection probability: P_D
- Gate probability: P_G
- Predicted measurement density of j th target: $p_{t|t-1}^{(j)}(y)$.

In the KF case:

$$p_{t|t-1}^{(j)}(y) = \mathcal{N}(y; \hat{y}_{t|t-1}^{(j)}, S_{t|t-1}^{(j)})$$

Hypothesis Probabilities: false alarm

False alarm (FA)

- Number of false alarms, m_t^{FA} , in V is distributed as:

$$P_{\text{FA}}(m_t^{\text{FA}}) = \frac{(\beta_{\text{FA}} V)^{m_t^{\text{FA}}} e^{-\beta_{\text{FA}} V}}{m_t^{\text{FA}}!}$$

- False alarm spatial density is $p_{\text{FA}}(y) = 1/V$

Hypothesis Probabilities: new track

New Target (NT)

- Number of new targets, m_t^{NT} is distributed as

$$P_{\text{NT}}(m_t^{\text{NT}}) = \frac{(\beta_{\text{NT}} V)^{m_t^{\text{NT}}} e^{-\beta_{\text{NT}} V}}{m_t^{\text{NT}}!}$$

- New target spatial density is $p_{\text{NT}}(y) = 1/V$

Hypothesis Probabilities: FA and NT

Let \mathcal{J}^{FA} be the set of false alarms (with m_t^{FA} elements), then

$$\Pr(\mathcal{J}^{\text{FA}} \text{ are the FA}) = m_t^{\text{FA}}! P_{\text{FA}}(m_t^{\text{FA}}) \prod_{i \in \mathcal{J}^{\text{FA}}} p_{\text{FA}}(y_t^{(i)}).$$

The FA are unordered, hence $m_t^{\text{FA}}!$ compensates for all the FA association possibilities. Insert Poisson distributed clutter uniformly in the tracking volume:

$$\Pr(\mathcal{J}^{\text{FA}} \text{ are the FA}) = m_t^{\text{FA}}! \frac{(\beta_{\text{FA}} V)^{m_t^{\text{FA}}} e^{-\beta_{\text{FA}} V}}{m_t^{\text{FA}}!} \frac{1}{V^{m_t^{\text{FA}}}} = (\beta_{\text{FA}})^{m_t^{\text{FA}}} e^{-\beta_{\text{FA}} V} \propto (\beta_{\text{FA}})^{m_t^{\text{FA}}}$$

The NT case follows analogously.

Hypothesis Probabilities: putting it all together (1/2)

Consider association hypothesis θ_t in measurement scan Y_t .

$$P(\theta_t|Y_t) \propto (\beta_{FA})^{m_t^{FA}} (\beta_{NT})^{m_t^{NT}} \left[\prod_{j \in \mathcal{J}} P_D P_G p_{t|t-1}^{(j)}(y_t^{(\theta_t^{-1}(j))}) \right] \left[\prod_{j \in \bar{\mathcal{J}}} (1 - P_D P_G) \right],$$

where

- \mathcal{J} is the set of indices of detected tracks (assigned).
- $\bar{\mathcal{J}}$ is the set of indices of non-detected tracks (not assigned).
- $\theta_t^{-1}(j)$ is the index of the measurement that is assigned to track $j \in \mathcal{J}$.
($\theta_t^{-1}(j) = \emptyset$ is shorthand for no measurement associated with track j .)
- All but the last factors are associated with a measurement.

Hypothesis Probabilities: putting it all together (2/2)

The association can be simplified, such that it computed as a combination of individual measurement contributions:

$$\begin{aligned} P(\theta_t|Y_t) &\propto (\beta_{FA})^{m_t^{FA}} (\beta_{NT})^{m_t^{NT}} \left[\prod_{j \in \mathcal{J}} P_D P_G p_{t|t-1}^{(j)}(y_t^{(\theta_t^{-1}(j))}) \right] \left[\prod_{j \in \bar{\mathcal{J}}} (1 - P_D P_G) \right] \\ &= \beta_{FA}^{m_t^{FA}} \beta_{NT}^{m_t^{NT}} \left[\prod_{j \in \mathcal{J}} \frac{P_D P_G p_{t|t-1}^{(j)}(y_t^{(\theta_t^{-1}(j))})}{(1 - P_D P_G)} \right] \left[\prod_{j \in \bar{\mathcal{J}} \cup \mathcal{J}} (1 - P_D P_G) \right] \\ &= \beta_{FA}^{m_t^{FA}} \beta_{NT}^{m_t^{NT}} \left[\prod_{j \in \mathcal{J}} \frac{P_D P_G p_{t|t-1}^{(j)}(y_t^{(\theta_t^{-1}(j))})}{(1 - P_D P_G)} \right] (1 - P_D P_G)^{m_t} \\ &\propto \beta_{FA}^{m_t^{FA}} \beta_{NT}^{m_t^{NT}} \left[\prod_{j \in \mathcal{J}} \frac{P_D P_G p_{t|t-1}^{(j)}(y_t^{(\theta_t^{-1}(j))})}{(1 - P_D P_G)} \right] \end{aligned}$$

Hypothesis Probabilities: final logarithmic expression

Global logarithmic association probability

$$\log P(\theta_t|Y_t) = m_t^{FA} \log \beta_{FA} + m_t^{NT} \log \beta_{NT} + \sum_{j \in \mathcal{J}} \log \frac{P_D P_G p_{t|t-1}^{(j)}(y_t^{(\theta_t^{-1}(j))})}{(1 - P_D P_G)}$$

Properties:

- One term per measurement
- The best association hence boils down to picking the right contribution from each measurement, in a consistent way

Assignment Matrix

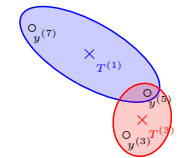
The assignment matrix organizes the possible measurement contributions to $\log p(\theta_t|Y_t)$ in an efficient way.

	$T^{(1)}$	$T^{(3)}$	$FA^{(3)}$	$FA^{(5)}$	$FA^{(7)}$	$NT^{(3)}$	$NT^{(5)}$	$NT^{(7)}$
$y_t^{(3)}$	$-\infty$	ℓ_{33}	$\log \beta_{FA}$	$-\infty$	$-\infty$	$\log \beta_{NT}$	$-\infty$	$-\infty$
$y_t^{(5)}$	ℓ_{51}	ℓ_{53}	$-\infty$	$\log \beta_{FA}$	$-\infty$	$-\infty$	$\log \beta_{NT}$	$-\infty$
$y_t^{(7)}$	ℓ_{71}	$-\infty$	$-\infty$	$-\infty$	$\log \beta_{FA}$	$-\infty$	$-\infty$	$\log \beta_{NT}$

Association matrix, $\mathcal{A}^{(1)}$

- The gain from assigning measurement $y^{(i)}$ to track $T^{(j)}$ is

$$\ell_{ij} = \log \frac{P_D P_G p_{t|t-1}^{(j)}(y_t^{(i)})}{(1 - P_D P_G)}.$$



	$T^{(1)}$	$T^{(3)}$
$y^{(3)}$	0	1
$y^{(5)}$	1	1
$y^{(7)}$	1	0

Validation matrix, $\mathcal{V}^{(1)}$

Assignment Problem

Assume a scan with m measurements and n “track hypotheses” (TC, FA, NT).

- Given the matrix $\mathcal{A} \in \mathbb{R}^{m \times n}$ with $n \geq m$.
- Define the binary matrix $Z = [z_{ij}]$, with $z_{ij} \in \{0, 1\}$.

Problem definition

$$\begin{aligned} \text{maximize:} \quad & \sum_{i,j} z_{ij} \mathcal{A}_{ij} \\ \text{subject to:} \quad & \sum_j z_{ij} = 1 \quad \forall i \quad (\dagger) \\ & \sum_i z_{ij} \leq 1 \quad \forall j \quad (\ddagger) \end{aligned}$$

† Each measurement is associated with exactly one track/FA/NT.

‡ Each track/FA/NT is associated with at most one measurement.

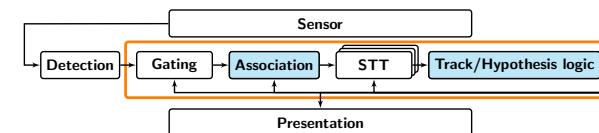
Assignment Problem: algorithms

- First considered in economics.
- For smaller problems an exhaustive search is possible, but this is inefficient.
- Earlier methods used linear programming techniques, like the Hungarian method which is computationally costly.

Assignment Problem: famous solutions

- **Munkres** algorithm obtains an optimal solution to the GNN assignment problem. An optimal solution minimizes the total cost of the assignments.
- **Auction** algorithm (by Bertsekas) finds a suboptimal solution to the GNN assignment problem by minimizing the total cost of assignment. While suboptimal, the auction algorithm is faster than the Munkres algorithm for large GNN assignment problems, for example, when there are more than 50 rows and columns in the cost matrix.
- **JVC** algorithm (by Jonker and Volgenant) solves the GNN assignment in two phases: begin with the auction algorithm and end with the Dijkstra shortest path algorithm.

Global Nearest Neighbor Tracker



Global Nearest Neighbor (GNN)

In each scan:

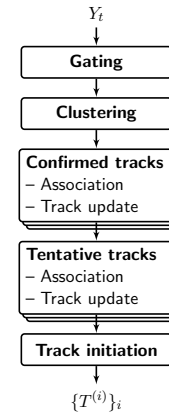
- Select the best association hypothesis, θ_t .
- Given θ_t :
 - Update all tracks with the associated measurement (usually using an EKF).
 - Update the track logic.
- Initiate new tracks from NT measurements.

Note on NT and FA handling

In the above steps, NT or FA does not matter, until the last step where anyhow all unassociated measurements should be given the chance to start up a new track.

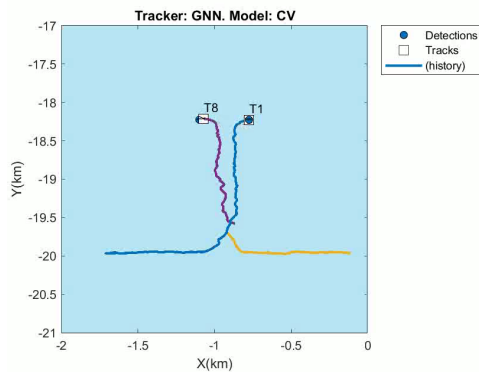
Introduce **external sources** (EX) combining FA and NT. EX becomes Poisson distributed with $\beta_{EX} = \beta_{FA} + \beta_{NT}$.

Global Nearest Neighbor (GNN): implementation details



- Apply gating and clustering to minimize the computational complexity.
- Use the EX trick to simplify the assignment problem further.
- Combine the tracking filter and target logic in one structure.
- Have separate containers for confirmed and tentative tracks.

MTT: GNN CV-model



<https://youtu.be/WFA2z-kr1vg>

- *Global nearest neighbor* (GNN) tracker
- Simple *constant velocity* (CV) model
- Note the label switch and that one of the tracks is lost half way, and restarted as a new one.

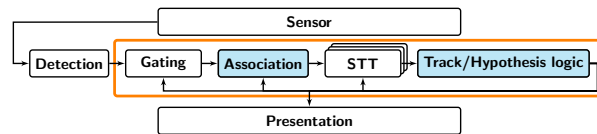
Global Nearest Neighbor: properties

- Makes a hard association decision:
 - + Optimal when the correct association is made.
 - Could break down completely with the wrong association.
- Works well when targets are well separated!
- Should not be used with poorly separated targets.
- Heavy clutter and low P_D could cause problems.
- Relatively fast and easy to implement.
- Works directly with the track logic discussed earlier.

Joint Probabilistic Data Association (JPDA) Filter

- JPDA is the soft decision equivalent of GNN in the way that PDA is a soft version of NN.
- All past is summarized with a single merged hypothesis.
- The number of targets is assumed fixed in the association, hence no NT associations in the possible hypotheses.
- A separate track initiation logic must run along with JPDAF to detect and initiate new tracks.

JPDA

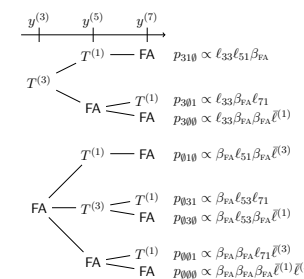


Joint Probabilistic Data Association (JPDA) Filter: details

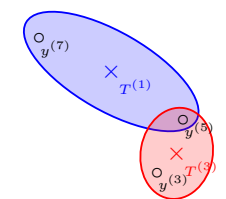
- All measurement associations are combined weighted with their likelihood of being true.
- For each previously established target, we need to calculate:
 - $P(\theta^{-1}(j) = i)$: Track T_j is associated with measurement $y^{(i)}$.
 - $P(\theta^{-1}(j) = \emptyset)$: shorthand for no measurement is associated with T_j .
- For measurement $y_t^{(i)}$ in the gate, the update is then made using the PDA update formulas with slightly modified probabilities to account for global association consistency.

Joint Probabilistic Data Association: probabilities (1/2)

Enumerate all possible measurement hypotheses and compute their respective likelihood. This can be done for each cluster independently.



- $\ell_{ij} = P_D p_{t|t-1}^{(j)}(y_t^{(i)})$
- $\bar{\ell}_j = 1 - P_D P_G$

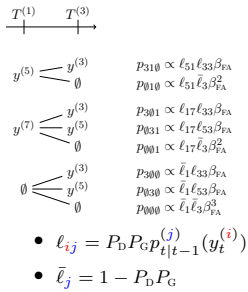


	$T^{(1)}$	$T^{(3)}$	$FA^{(3)}$	$FA^{(5)}$	$FA^{(7)}$
$y_t^{(3)}$	$-\infty$	ℓ_{33}	$\log \beta_{FA}$	$-\infty$	$-\infty$
$y_t^{(5)}$	ℓ_{51}	ℓ_{53}	$-\infty$	$\log \beta_{FA}$	$-\infty$
$y_t^{(7)}$	ℓ_{71}	$-\infty$	$-\infty$	$-\infty$	$\log \beta_{FA}$

Association matrix, $\mathcal{A}^{(1)}$

Joint Probabilistic Data Association: probabilities (2/2)

Rearrange the hypotheses to be able to compute the probability for each separate track.



$$\Pr(\theta^{-1}(1) = 5) = \frac{1}{C} (p_{310} + p_{010})$$

$$\Pr(\theta^{-1}(1) = 7) = \frac{1}{C} (p_{301} + p_{031} + p_{001})$$

$$\Pr(\theta^{-1}(1) = 0) = \frac{1}{C} (p_{300} + p_{030} + p_{000})$$

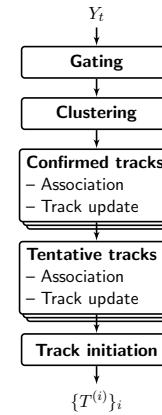
$$\Pr(\theta^{-1}(3) = 3) = \frac{1}{C} (p_{310} + p_{301} + p_{300})$$

$$\Pr(\theta^{-1}(3) = 5) = \frac{1}{C} (p_{031} + p_{030})$$

$$\Pr(\theta^{-1}(3) = 0) = \frac{1}{C} (p_{010} + p_{001} + p_{000})$$

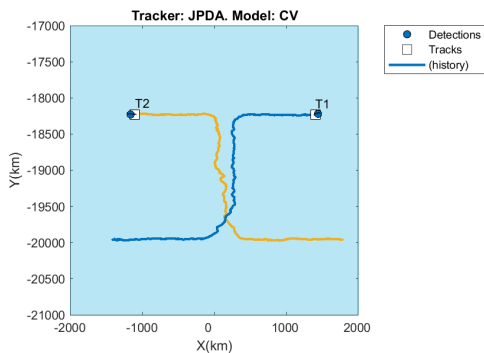
$$C = \sum_i p_i$$

Joint Probabilistic Data Association: details



- For each cluster, calculate probabilities for each target in the cluster by using a hypothesis tree.
- Use the targets PDA equivalent measurement for the update (see lecture 3).
- Unused measurements are used to initiate new tracks.
- Promote track status according to standard track logic.

MTT: JPDA CV-model



https://youtu.be/_YBSJw4P0rY

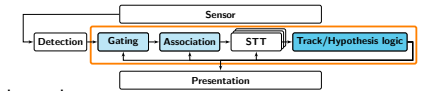
- *Joint probabilistic data association (JPDA)* tracker
- Simple *constant velocity (CV)* model
- Note that the label switch, but there are no lost tracks.

Joint Probabilistic Data Association: properties

- Makes no hard association decision:
 - + More robust in heavily cluttered environments with low P_D .
 - Sub-optimal compared to using the correct associations.
- Works well when targets are well separated!
- Closely separated targets suffer from coalescence, *i.e.*, neighboring tracks become identical.
- More complicated and more computationally complex than GNN.
- Consideration required when implementing the track logic.

Summary

Summary



- Extended previous methods to several targets.
- Methods for gating, clustering, and association were presented, yielding the validation and association matrix.
- SHT: *One* measurement association hypothesis is used
 - GNN: A hard decision; choose the most likely association hypothesis.
The association problem can be solved with many of-the-shelf algorithms, e.g., auction, after constructing the association (cost) matrix.
 - JPDA: A soft decision; marginalize all possible associations.
How to combine the possible measurements depends on the association matrix.

Exercises

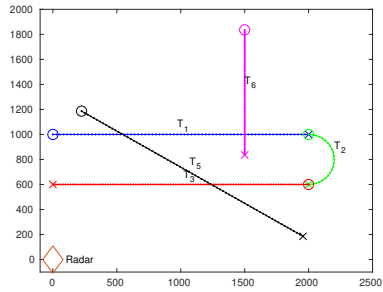
Lecture Schedule (preliminary)

Le	Topic	Date	Ex
1	Introduction	Sept 16	15–16
1b	Preliminaries	Sept 16	16–17
2	Models for Target tracking	Sept 19	13–15
3	Single target tracking	Sept 25	15–17 Ex 1
4	Multi-target tracking (1/2): GNN, JPDA	Oct 16	13-15 Ex 2
5	Multi-target tracking (2/2): MHT	Oct 30	10-12 Ex 3
6	Random Finite Sets: PHD, etc	Nov 14	13-15
7	Random Finite Sets: PHD, etc part 2	Nov 27	15-17
8	Various topics (TkBD, T2T, ETT)	Fall	
9	Ethical aspects	Fall	
	Guest lecture	Fall	

- Lectures are in **Large conference room in Visionen**, unless otherwise stated.
- Exercises are due at the end of the course.
(Doing them as the course progresses is **highly** recommended!)
- Dates are preliminary, check homepage and e-mail for updates.

Exercise 2

1. Simulate a more complicated scenario, with several targets:



- Simulate trajectory
- Generate measurement:
 - P_D
 - P_{FA}
 - clutter
- Details specified in the exercise

Exercise 2

2. MTT: GNN and JPDA

- In the exercise a detailed step-by-step instruction is given on how to build a MTT for GNN/JPDA.
- Apply the measurements to a GNN-tracker (a MATLAB version of the auction algorithm is given)
- Apply the measurements to a JPDA-tracker (MATLAB code to compute the measurement to track probabilities is available)

3. MTT: mysterious data

- At the end a mysterious data set is given without ground truth. Apply your GNN and JPDA implementations to extract the targets.

