

Target Tracking

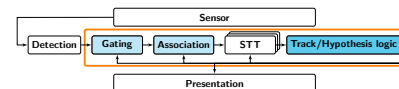
Le 8: RFS tracking

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Summary: lecture 7



Various methods in MTT

- Performance evaluation using OSPA
- Track to Track fusion
- Track Before Detect
- Extended Target Tracking (ETT)
- Group Tracking

We will now leave the classical MTT for an alternative set representation.

References on Random Finite Set Methods: general

- B.-N. Vo, M. Mallick, Y. Bar-Shalom, S. Coraluppi, R. Osborne, III, R. Mahler, and B.-T. Vo. *Multitarget Tracking*. Wiley Encyclopedia of Electrical and Electronics Engineering, 2015.
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References on Random Finite Set Methods: PHD filters

- B.-N. Vo and W.-K. Ma. [The Gaussian mixture probability hypothesis density filter.](#) *IEEE Transactions on Signal Processing*, 54(11):4091–4104, 2006.
doi: [10.1109/TSP.2006.881190](https://doi.org/10.1109/TSP.2006.881190)
- D. Fränken, M. Schmidt, and M. Ulmke. ["Spooky action at a distance" in the cardinalized probability hypothesis density filter.](#) *IEEE Transactions on Aerospace and Electronic Systems*, 45(4):1657–1664, 2009.
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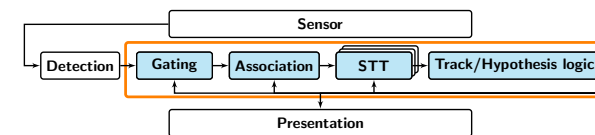
References on Random Finite Set Methods: LMB filters

- S. Reuter, B.-T. Vo, B.-N. Vo, and K. Dietmayer. [The labeled multi-Bernoulli filter.](#) *IEEE Transactions on Signal Processing*, 62(12):3246–3260, 2014
- B.-N. Vo, B.-T. Vo, and D. Phung. [Labeled random finite sets and the Bayes multi-target tracking filter.](#) *IEEE Transactions on Signal Processing*, 62(24):6554–6567, 2014

References on Random Finite Set Methods: PMBM filters

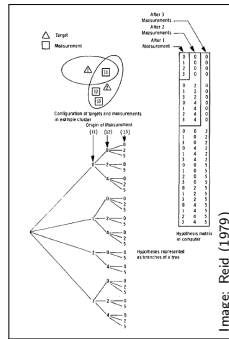
- J. Williams. [Marginal multi-Bernoulli filters: RFS derivation of MHT, JIPDA, and association-based MeMBer.](#) *IEEE Transactions on Aerospace and Electronic Systems*, 51(3):1664–1687, July 2015
- Á. F. García-Fernández, J. L. Williams, K. Granström, and L. Svensson. [Poisson multi-Bernoulli mixture filter: Direct derivation and implementation.](#) *IEEE Transactions on Aerospace and Electronic Systems*, 54(4):1883–1901, 2018

Random Finite Set (RFS)

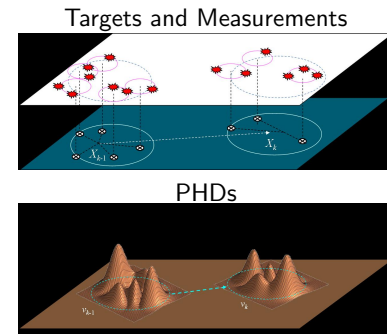


Properties of Classic Tracking Methods

- State of the art: Multi-hypothesis tracker (MHT)
- Combines several more or less independent components:
 - Single target tracking
 - Association
 - Track creation, maintenance, deletion
 - Outliers
- No common mathematical formulation for all components.
- Separate methods for complexity reduction.



Desired Tracking Method Properties



Figures obtained from random set filtering website
<http://randomsets.eps.hv.ac.uk/>

- One unified mathematical formulation.
- Solve the problem with methods that integrate all parts in one.
- Complexity reductions as approximations to mathematical formulation, instead of engineering solutions.

Tracking Assumptions

- Number of targets, unknown and varies over time.
- Origin of observations, unknown.
- Independent measurements (independent measurement noise).
- Independent motion (targets do not influence each other).
- Each target produces at most one observation (point target assumption), and each observation stem from at most one target.
- Measurements can be missed and clutter exist.

Tracking Problem Seen as a Set Problem

- Should describe the same fundamental problem the classic method, and fulfill the same assumptions.
- The targets in a scene, can be seen as a **set of tracks** with unknown cardinality (number of elements in the set).
- The observations in a scan is a **set of measurements**.
- **Sets are unordered**. This has implications for track labeling and the association process.
- The task is to estimate the cardinality, and the state of each track, both assumed stochastic, using Bayesian methods.

Solutions Using the Set Formulation

- Random finite sets (RFSs) describes sets of random elements and random cardinality.
- RFS based approaches propagate the posterior density of the multi-target state recursively in time, hence describing the complete tracking problem.
- Several different approximations of this exist:
 - Probability Hypothesis Density (PHD): propagate first moment
 - **Cardinalized PHD (CPHD)**: also propagate the cardinality
 - Multi-Bernoulli filters: propagates the parameters of a multi-Bernoulli distribution that approximate the posterior multi-target density
 - Poisson Multi-Bernoulli Mixture (PMBM): propagates unobserved targets as a PHD and observed targets as a MB mixture.
- Labeling: sometimes disregarded in these methods, but can be added (target id).

RFS Preliminaries

Random Finite Sets (RFS)

Examples:

- \emptyset , $\{3.14\}$, $\{2.7, 9.82\}$, examples of a RFS of real numbers.
- Let $x_k^i \in \mathbb{R}^{n_x}$ for $i = 1, \dots, \infty$. Then, some realizations X of the random variable \mathbf{X} can be \emptyset , $\{x_k^1\}$, $\{x_k^1, x_k^2\}$, $\{x_k^1, x_k^2, x_k^3\}$, \dots
- **Note:** $\{x_k^2, x_k^1\}$ is the same as $\{x_k^1, x_k^2\}$ as sets are unordered.
- Y_t all observations obtained in the scan at time t .
- X_t all targets at time t .

Definition: random finite set (RFS)

A random finite set \mathbf{X} is a random variable that has realizations in the form $\mathbf{X} = X \in \mathcal{S}$ where \mathcal{S} is the set of all finite subsets of some underlying space \mathbb{S} .

Random Sets: connections to normal random stochastic variables

Random finite sets

- Every element is a *random stochastic variables* (RSV), with a PDF.
- The cardinality is a discrete positive RSV, with a PDF.

Random finite sets properties

- It is possible to compute the probability of a certain instance, $\Pr(X)$.
- The *belief mass* function has the same purpose as the PDF of normal RSV, but does not sum to 1 but instead the cardinality of the RFS.
- It is possible to integrate over RFS $\int p(X) \delta X$, with “minor” modifications to how the integral is computed.

Random Sets: connections to normal random stochastic variables

It is possible to define a PDF for the RFS (with a slight abuse of notation):

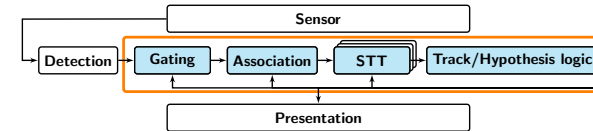
$$p(\{x^{(1)}, x^{(2)}, \dots, x^{(n)}\} | n) = n! p(x^{(1)}, x^{(2)}, \dots, x^{(n)})$$

Hence, we also need to define integrals over sets.

Definition (Set Integral)

$$\int p(X) \delta X = p(\emptyset) + \int p(x^{(1)}) dx^{(1)} + \frac{1}{2!} \int p(x^{(1)}, x^{(2)}) dx^{(1)} dx^{(2)} + \frac{1}{3!} \int p(x^{(1)}, x^{(2)}, x^{(3)}) dx^{(1)} dx^{(2)} dx^{(3)} + \dots$$

Generic RFS Filter



General Bayesian Filtering Solution

With these tools, we want to approximate the Bayesian filtering solution,

$$p(x_t | \mathbb{Y}_{t-1}) = \int p(x_t | x_{t-1}) p(x_{t-1} | \mathbb{Y}_{t-1}) dx_{t-1} \quad (\text{TU})$$

$$p(x_t | \mathbb{Y}_t) = \frac{p(y_t | x_t) p(x_t | \mathbb{Y}_{t-1})}{p(y_t | \mathbb{Y}_{t-1})}. \quad (\text{MU})$$

In this case with RFSs as input and states (with slight abuse of notation):

$$p(X_t | \mathbb{Y}_{t-1}) = \int p(X_t | X_{t-1}) p(X_{t-1} | \mathbb{Y}_{t-1}) \delta X_{t-1} \quad (\text{TU-RFS})$$

$$p(X_t | \mathbb{Y}_t) = \frac{p(Y_t | X_t) p(X_t | \mathbb{Y}_{t-1})}{p(Y_t | \mathbb{Y}_{t-1})}. \quad (\text{MU-RFS})$$

In principal the solutions are very similar, but due to technicalities, the implementation is quite different.

RFS Models: dynamic model

- Denote the set of targets X_t at time t .
- X_t encodes the number of targets, and their positions.
- As $x_t = f(x_{t-1}, w_t)$ describes how targets propagate in time,

$$X_t = F(X_{t-1}) \cup W_t$$

describes how the RFS X_t evolves over time.

- F yields a RFS, in which each existing target in X_{t-1} has been either been propagated using f (implicitly affected by process noise), or died.
- W_t is a RFS with targets born at time t .

RFS Models: measurement model

- Denote the set of measurements, in the scan at time t , Y_t .
- The equivalent of $y_k = h(x_k) + e_k$ is

$$Y_k = H(X_k) \cup V_k$$

which is the RFS of all measurements procuded at time t .

- H yields a RFS with the measurements are generated from the targets in X_k using $h(x)$ (implicitly assuming measurement noise). A measurement is generated from at target with probability P_D , and all measurements are affected by measurement noise.
- The RFS V_k contains all clutter/false measurements.

RFS Filtering: Bayesian solution

- With these definitions, its possible to compute

$$p(X_t | \mathbb{Y}_{t-1}) = \int p(X_t | X_{t-1}) p(X_{t-1} | \mathbb{Y}_{t-1}) \delta X_{t-1} \quad (\text{TU-RFS})$$

$$p(X_t | \mathbb{Y}_t) = \frac{p(Y_t | X_t) p(X_t | \mathbb{Y}_{t-1})}{p(Y_t | \mathbb{Y}_{t-1})}. \quad (\text{MU-RFS})$$

where a **set integral** is needed.

- This filter is computationally prohibitive to implement except few cases.

Important difference

The moments, as heavily used in, e.g., Kalman filter,

~~$$\hat{X}_k = \int X_k p(X_k | Y_{0:k}) \delta X_k = E(X_k | Y_{0:k})$$~~

is **not** well-defined! (How are elements with different cardinality combined?)

Random Finite Set Descriptions: Poisson point process

RFS can be represented in many different ways, which in turn leads to different approximations, resulting in different filter algorithms.

Poisson Point Process (PPP) RFS

$$p(X) = e^{-\int \lambda(x) dx} \prod_{x \in X} \lambda(x)$$

Typical usage:

- Targets
- Clutter
- Appearing objects
- Measurements from extended targets

Random Finite Set Descriptions: multi-Bernoulli RFS

RFS can be represented in many different ways, which in turn leads to different filter approximations.

Bernoulli RFS

$$p(X) = \begin{cases} 1 - r, & X = \emptyset \\ rp(x), & X = \{x\} \end{cases}$$

- It can be shown that cardinality probability is $\rho(0) = 1 - r$, $\rho(1) = r$, and $\rho(n > 1) = 0$.

Multi-Bernoulli RFS

$$p(\{x_1, \dots, x_n\}) = \prod_{j=1}^M (1 - r^{(j)}) \sum_{1 \leq i_1 \neq \dots \neq i_n \leq M} \prod_{j=1}^n \frac{r^{(i_j)} p^{(i_j)}(x_j)}{1 - r^{(i_j)}}$$

$$\rho(n) = \prod_{j=1}^M (1 - r^{(j)}) \sum_{1 \leq i_1 \neq \dots \neq i_n \leq M} \prod_{j=1}^n \frac{r^{(i_j)}}{1 - r^{(i_j)}}$$

- Union of several independent Bernoulli RFS.
- p_i is the belief function for the Bernoulli RFSs.
- $\rho(n)$ is the cardinality probability.

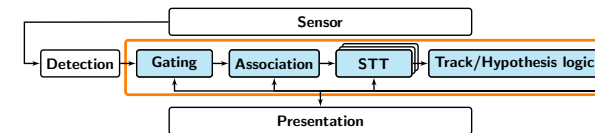
Relations Between PPP and MB

Note

- A multi-Bernoulli RFS can be approximated by a Poisson point process if the existence probabilities are $r_i < 0.1$.
- Any Poisson point process can be approximated by a multi-Bernoulli RFS, but it may require a large number of Bernoulli components, N .

A Poisson point process is often less computationally expensive than a multi-Bernoulli RFS. However, in a Poisson point process both the mean and the variance of the cardinality is λ , hence the uncertainty grows with the number of components.

Probability Hypothesis Density (PHD) Filter



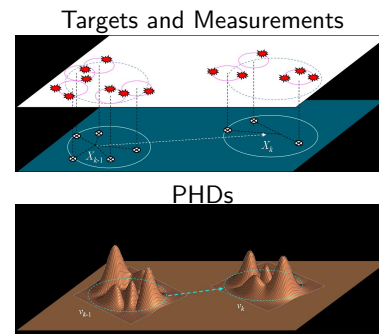
Probability Hypothesis Density (PHD)

- Suppose $X_t = \{x_t^{(1)}, x_t^{(2)}, \dots, x_t^{(n)}\}$. Define a scalar valued function from X_t that can be summed,

$$h_{X_t}(x) = \sum_{i=1}^n \delta_{x_t^{(i)}}(x).$$

- Then, *the probability hypothesis density (PHD)* is the expectation of $h_{X_t}(x)$ with respect to X_t .

$$\text{PHD}_{t|t}(x) = E(h_{X_t}(x)|Y_{0:t})$$



Figures obtained from random set filtering website
<http://randomsets.eps.liv.ac.uk/>

PHD Filter: PHD definition

A useful definition uses the expected mean of a function (converting sets to vectors) and with $T_{X \cup X'} = T_X + T_{X'}$ if $X \cap X' = \emptyset$.

Commonly used:

$$E[T] = \int T_X p(X) \delta X$$

where the transformation is the Dirac density, so $T_X = \delta_X$, i.e., $\delta_X(x) = 0$ if $X = \emptyset$, otherwise $\delta_X(x) = \sum_{w \in X} \delta_w(x)$.

This definition yields the *probability hypothesis density (PHD)* (intensity function):

$$D(x) = \int \delta_X(x) p(X) \delta X.$$

PHD Filter: assumptions

PHD: Model Assumptions

- Motion model PDF: $p_{t+1|t}(x|x')$
- Survival probability for existing targets: $p_{S,t+1|t}(x')$
- Spawning of new targets from existing: $b_{t+1|t}(X|x')$
- Appearance of new targets: $b_{t+1}(X)$
- Probability of detection: p_D
- False alarm model (Poisson distribution): probability $c(x)$
- Single target likelihood: $L(x) = p(y|x)$

PHD Filter: filter recursion (1/2)

Actions can be formulated as a RFS, where the possibilities are: survival, spawning of existing target, and spontaneous birth.

$$X_t = \left[\bigcup_{x \in X_{t-1}} p_{t|t-1}(x'|x) \right] \cup \left[\bigcup_{x \in X_{t-1}} b_{t|t-1}(x) \right] \cup b_t$$

Given this, its possible to derive the filter recursion with a time and measurement updates.

PHD Filter: filter recursion (2/2)

The PHD filter recursions

$$D_{t+1|t}(x) = \int \underbrace{(p_S(x')p_{t+1|t}(x|x'))}_{\text{Survival}} + \underbrace{b_{t+1|t}(x|x')}_{\text{Spawned}} D_{t|t}(x') dx' + \underbrace{b_{t+1}(x)}_{\text{Birth}}$$

$$D_{t|t}(x) = (1 - p_D(x))D_{t|t-1}(x) + \sum_{y \in Y_t} \frac{p_D(x)p(y|x)D_{t|t-1}(x)}{c(y) + \int p_D(x')p(y|x')D_{t|t-1}(x')dx'}$$

Note:

The first-order moment density (or intensity) is similar to a PDF, but integrates to the number of targets instead of 1!

The GM-PHD Filter

To implement the PHD idea, the inherent exponential complexity must be handled. One way is to use a Gaussian sum filter bank with pruning and merging, yielding the *Gaussian mixture PHD* (GM-PHD).

- Assume p_S and p_D are state-independent (to simplify things).
- Assume that $b_{t+1|t}(x|x')$ and $b_{t+1}(x)$ are **Gaussian mixtures**.

For details see for instance Vo and Ma (2006) or Hendeby and Karlsson (2014).

The GM-PHD filter uses the following PHD representation

$$v_{t|t}(x) = \sum_{i=1}^{J_{t|t}} w_{i,t}^{(t)} \mathcal{N}(x; m_{i,t}^{(t)}, P_{i,t}^{(t)}) \quad (5)$$

Given that the tracking problem can be modeled using constant $p_{D,i}$, a Gaussian-mixture birth process

$$\gamma_t(x) = \sum_{i=1}^{J_{t|t}} w_{i,t}^{(t)} \mathcal{N}(x; m_{i,t}^{(t)}, P_{i,t}^{(t)}) \quad (6a)$$

and a Gaussian-mixture spawning process

$$\beta_{t|t-1}(x) = \sum_{i=1}^{J_{t|t-1}} w_{i,t|t-1} \mathcal{N}(x; F_{i,t|t-1} + c + d_{i,t|t-1}^{(t)}, Q_{i,t|t-1}^{(t)}) \quad (6b)$$

the PHD update are given by the following expressions. Starting with the filtered PHD, $v_{t-1|t-1}$ in (5) the PHD time update is given by

$$v_{t|t-1}(x) = v_{S,t|t-1}(x) + v_{B,t|t-1}(x) + \gamma_t(x) \quad (7)$$

where $v_{S,t|t-1}(x)$ is the PHD of surviving targets, $v_{B,t|t-1}(x)$ the PHD of new targets spawned from existing ones in this time update, and $\gamma_t(x)$ the PHD of newly born targets, as defined below.

The surviving target PHD is given by

$$v_{S,t|t-1}(x) = \sum_{i=1}^{J_{t|t-1}} w_{i,t|t-1}^{(t)} \mathcal{N}(x; m_{i,t|t-1}, P_{i,t|t-1}) \quad (8a)$$

where

$$w_{i,t|t-1}^{(t)} = p_{S,i}^{(t)} w_{i,t-1}^{(t-1)} \quad (8b)$$

$$m_{i,t|t-1}^{(t)} = F_{i,t|t-1} + m_{i,t-1}^{(t-1)} \quad (8c)$$

$$P_{i,t|t-1}^{(t)} = F_{i,t|t-1} + P_{i,t-1}^{(t-1)} + Q_{i,t-1} \quad (8d)$$

and $p_{S,i}^{(t)}$ is the probability that a target in time $t-1$ survives to time t .

The spawned target PHD is given by

$$v_{B,t|t-1}(x) = \sum_{i=1}^{J_{t|t-1}} \sum_{j=1}^{J_{i,t|t-1}} w_{i,j,t|t-1}^{(t)} \mathcal{N}(x; m_{i,j,t|t-1}^{(t)}, P_{i,j,t|t-1}^{(t)}) \quad (9a)$$

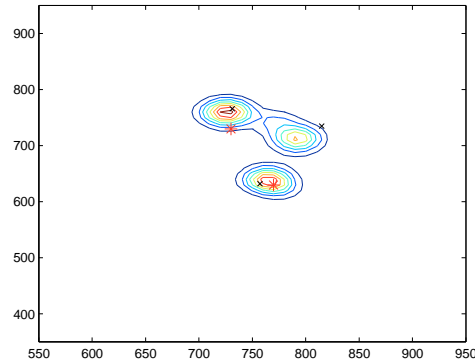
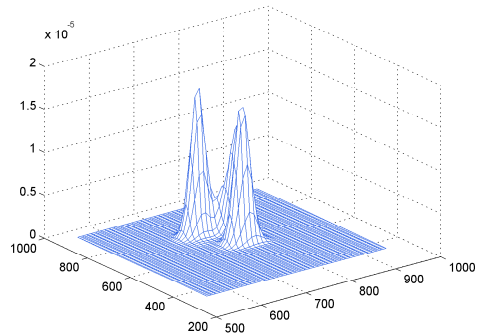
where

$$w_{i,j,t|t-1}^{(t)} = w_{i,t|t-1}^{(t-1)} w_{j,t|t-1}^{(t-1)} \quad (9b)$$

$$m_{i,j,t|t-1}^{(t)} = F_{i,t|t-1} + m_{j,t|t-1}^{(t-1)} + d_{i,j,t|t-1}^{(t)} \quad (9c)$$

The GM-PHD Filter: example

Illustration of PHD at time: $k + 1$



Note: The expected mean over the intensity function sums to the number of targets.

Remarks and Extensions

There exist several approximation/implementation ideas as well as other important issues for multi-target tracking using PHD:

- Birth at a fixed source (can be relaxed).
- No association, but pruning and mixing.
- There exist other PHD implementations, for instance one based on the particle filter.
- The cardinalized PHD (CPHD):

We want second order approximation, but instead of a full implementation try to improve upon existing PHD. The CPHD-filter will not fully model the second order moment but instead it propagates both the intensity $D_{t|t}(x)$ and the cardinal distribution $p_{t|t}(n)$. Hence, it propagates the entire probability density for the number of targets.

- PHD with track labeling.

Efficient Gaussian Sum Implementations

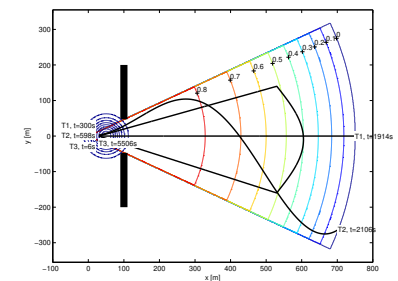
Given a Gaussian sum assumption of the PHD, this boils down to more or less a tracking filter where all associations are attempted and then merged in each time step. (Without considering target identity.)

PHD Filter Example: Tracking divers

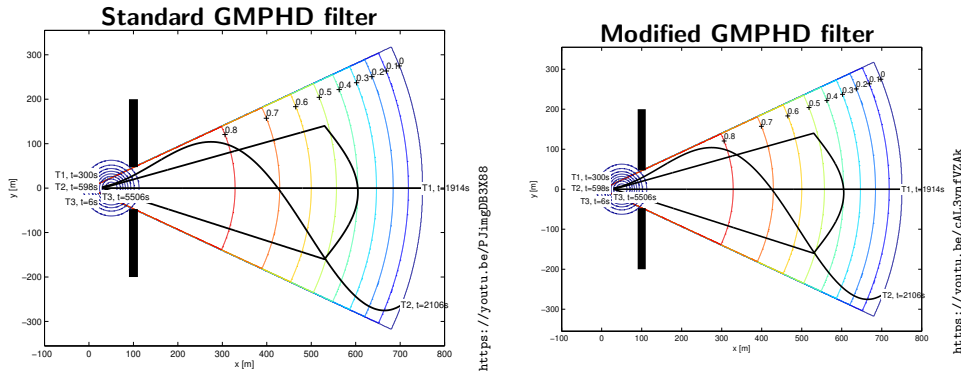
- Example of tracking divers using sonar.
- Modified GM-PHD to handle varying p_D ,

$$p_D(x) = 0.9 - 4 \cdot 10^{-7} R^2 - 1.6 \cdot 10^{-9} R^3,$$

- Fairly high clutter.
- For details:
Hendeby and Karlsson (2014)



PHD Filter Example



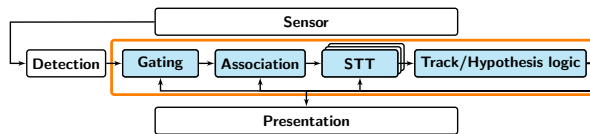
- Probability of detection dies off as a 3rd-degree polynomial, inspired by real data.

Cardinalized Probability Hypothesis Density (CPHD)

- In the PHD filter the number of target are captured by integrating the intensity function.
- The PHD can be seen as a first order moment filter.
- The CPHD is the equivalent to second order moment.
- CPHD: the cardinality is explicitly estimated in combination with the intensity.
- Motivations: why use CPHD instead of PHD?
 - The assumption of Poisson target cardinality makes the PHD sensitive to clutter.
 - “Spooky action at a distance” (Fränken et al., 2009): Missed measurements shifts the PHD from unrelated areas to detected parts.

The last years other methods are more important, for instance Multi-Bernoulli filters, and Poisson multi-Bernoulli mixture filters.

Multi-Bernoulli Filters



Labeled Multi-Bernoulli Representation

- Bernoulli RFS:

$$p(X) = \begin{cases} 1 - r & X = \emptyset \\ rp(x) & X = \{x\} \end{cases}$$

- Multi-Bernoulli RFS

$$p(\{x_1, \dots, x_n\}) = \prod_{j=1}^M (1 - r^{(j)}) \sum_{1 \leq i_1 \neq \dots, \neq i_n \leq M} \prod_{j=1}^n \frac{r^{(i_j)} p^{(i_j)}(x_j)}{1 - r^{(i_j)}}$$

$$\rho(n) = \prod_{j=1}^M (1 - r^{(j)}) \sum_{1 \leq i_1 \neq \dots, \neq i_n \leq M} \prod_{j=1}^n \frac{r^{(i_j)}}{1 - r^{(i_j)}}$$

- Labeled Multi-Bernoulli RFS

Add a **unique label** ℓ to the state, and the RFS becomes $\{(r^{(\ell)}, x^{(\ell)})\}_{\ell \in \mathcal{L}}$.

Labeled Multi-Bernoulli: algorithm (1/2)

Time update

$$p_+(X_+) = \int f(X_+)p(X)\delta X$$

$$p_+ = \{(r_{+,S}^{(\ell)}, p_{+,S}^{(\ell)})_{\ell \in \mathcal{L}} \cup \{(r_B^{(\ell)}, p_B^{(\ell)})_{\ell \in \mathcal{B}}\}$$

where

$$r_{+,S} = \eta_S(\ell)r^{(\ell)}$$

$$p_{+,S}^{(\ell)} = \frac{\int p_S(\xi, \ell)f(x|\xi, \ell)p(\xi|\ell) d\xi}{\eta_S(\ell)}$$

$$\eta_S(\ell) = \int p_S(\xi, \ell)p(\xi|\ell) d\xi$$

$\{(r_B^{(\ell)}, p_B^{(\ell)})_{\ell \in \mathcal{B}}\}$ is a birth model.

Labeled Multi-Bernoulli: algorithm (2/2)

Measurement update

$$p(\cdot|Z) \approx \bigcup_{i=1}^N \{(r^{(\ell,i)}, p^{(\ell,i)})\}_{\ell \in \mathcal{L}_+^{(i)}}$$

$$r^{(\ell,i)} = \sum_{(I_+, \theta) \in \mathcal{F}(\mathcal{L}_+^{(i)}) \times \Theta_{I_+}} \omega^{(I_+, \theta)}(Z^{(i)}) \mathbf{1}_{I_+}(\ell)$$

$$p^{(\ell,i)}(x) = \frac{1}{r^{(\ell,i)}} \sum_{(I_+, \theta) \in \mathcal{F}(\mathcal{L}_+^{(i)}) \times \Theta_{I_+}} \omega^{(I_+, \theta)}(Z^{(i)}) \mathbf{1}_{I_+}(\ell) p^{(\theta)}(x, \ell | Z^{(i)})$$

$$\omega^{(I_+, \theta)}(Z^{(i)}) \propto \omega_{+,i}^{(I_+)} [\eta_{Z^{(i)}}^{(\theta)}]^{I_+}$$

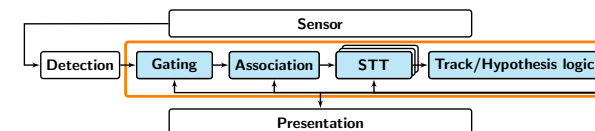
$$= \prod_{\ell \in \mathcal{L}_+^{(i)} - I_+} (1 - r_+^{\ell}) \prod_{\ell' \in I_+^a} r_+^{\ell'} \eta_{Z^{(i)}}^{(\theta)}(\ell') \prod_{\ell'' \in I_+^n} r_+^{\ell''} \eta_{Z^{(i)}}^{(\theta)}(\ell'')$$

Observation

The labeled Multi-Bernoulli filter is similar to the MHT

- All potential targets can be handled separately, extra book keeping for existence probabilities etc.
- The δ -generalized labeled multi-Bernoulli (δ -GLMB) filter, which essentially is a TO-MHT.

Poisson Multi-Bernoulli Mixture (PMBM) Filter

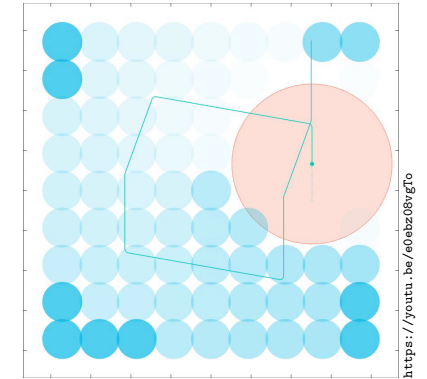


Poisson Multi-Bernoulli Mixture (PMBM) Filter

- Relatively new, Williams (2015).
- Can be considered the new state-of-the-art MTT method. (This can be debated.)
- Has one description of unknown and unobserved targets.
- Has one description of observed targets.
- The PMBM is a conjugate prior, hence suitable as a recursive filter.

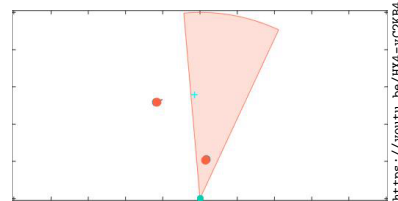
PMBM: unobserved targets

- Maintain a representation of unobserved targets.
- “P” in PMBM is a Poisson point process for unobserved targets.
- Often handled as a Gaussian mixture PHD, much similar to a GM-PHD filter but removing detected targets.
- Alternatives to GM-PHD exists, e.g., Boström-Rost et al. (2021).



PMBM: observed targets

- Observed targets are extracted from the unobserved PPP and handled separately.
- “MBM” in PMBM is a multi-Bernoulli mixture for observed targets.
- Basically a δ -generalized LMB (δ -GLMB) filter, which is a type of LMB with preferable properties.
- Can be efficiently implemented using a TO-MHT.



PMBM Algorithm

- Conceptually:
 - Run a PHD filter too keep track of unobserved targets.
 - Run a TO-MHT too keep track of observed targets.
 - When an unobserved target is observed, remove it from PHD filter, and insert into the MHT again.
 - Weights are update to make this correct.
- For details see, e.g., Williams (2015); García-Fernández et al. (2018)

Summary

- *Random finite set* (RFS) represents a “new” view of target tracking, but are mostly very similar to classic methods.
- Many different approximations, resulting in different filters:
 - *Probability hypothesis distribution* (PHD) filter
 - Multi-Bernoulli distribution filters
 - *Poisson multi-Bernoulli mixture* (PMBM) filter
- Utilize a random set formulation to include the full MTT problem in a single mathematical formulation.

Course Summary

- Focus on classic multi-target tracking problems, with outlooks to common extensions and the RFS formulation.
- **Examination** (how many intend to get credits?):
 - Exam, 2 ETCS credits:
Take home exam, 1 h exam. Tests the understanding of the principles discussed in the course.
Due: End of January (contact us to schedule the exam).
 - Exercises, 4 ETCS credits:
Show hands on experience of important discussed methods.
Due: December 19, 2021
 - Project, 3 ETCS credits:
More advanced utilization of MTT techniques, preferably related to your research.
Discuss details with us.