Target Tracking
Le 1: Introduction

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2 Multi-Target Tracking Overview
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Course Information

Multi-Target Tracking Course, Spring 2021

Aim

The aim of the course is to provide an introduction to *multi-target tracking* (MTT); both theoretical and practical aspects. After the course a student should be able to explain the basic ideas underlying MTT and feel confident to implement the fundamental methods.

Course activities:

- 7 lectures where the theoretical aspects of MTT are explained.
- 1 guest lecture (pending on the pandemic development).
- Practical coding exercises, performed on your own.

Responsible:

- Gustaf Hendeby (gustaf.hendeby@liu.se)
- Rickard Karlsson (rickard.g.karlsson@liu.se)

Course homepage:

- `https://mtt.edu.hendeby.se`

Course Content

- Single-target tracking (STT)
- Motion and sensor models:
  - Common tracking models
  - Maneuvering targets (IMM)
  - Clutter
- Multi-target tracking (MTT):
  - Association
  - Track logic
  - Global Nearest Neighbor (GNN) Tracker
  - Multi-Hypotheses Tracker (MHT)
- Outlook, modern methods:
  - Track before detect (TkBD)
  - RFS/FISST: Probability hypothesis density (PHD), Multi-Bernoulli, Poisson multi-Bernoulli mixture (PMBM)
  - Track-to-track fusion (T2TF)

Course Examination

Three independent parts with different focuses:

1. Basic theory and understanding: exam (2 ETCS credits)

   Theory is examined in a brief written exam.

2. Implementation and practice: exercises (4 ETCS credits)

   Implementation skill and practical knowhow are examined using assignments during the course.

3. Research related work: project (3 ETCS credits)

   Use course skills extensions on the topic for a larger tracking project, preferably related to your research. Individually or in a group of two.

Course Prerequisites

Familiarity with:

- Basic knowledge of probability theory
- State-space models
- Bayesian estimation methods
  - Kalman filter (KF)
  - Extended Kalman filter (EKF)
  - Unscented Kalman filter (UKF)
  - Particle filter (PF)
- Coding in MATLAB or similar (for the exercises)

Suitable background material

- Sensor Fusion course (TSRT14): `http://www.control.isy.liu.se/student/tsrt14`
- Selected sensor fusion videos: `https://mtt.edu.hendeby.se/prerequisite.html`
- F. Gustafsson, L. Ljung, and M. Millnert. *Signal processing.* Studentlitteratur, 1. edition, 2010.
- F. Gustafsson. *Statistical Sensorfusion.* Studentlitteratur, 3. edition, 2018.
- T. Kailath, A. H. Sayed, and B. Hassibi. *Linear Estimation.* Prentice-Hall, Inc, 2000. ISBN 0-13-022464-2.
- S. M. Kay. *Fundamentals of Statistical Signal Processing: Estimation Theory*, volume 1. Prentice-Hall, Inc, 1993. ISBN 0-13-042268-1.

Lecture Schedule (preliminary)

Le	Topic	Date		Ex
1	Introduction	Sept 15	13–15	
2	Models for Target tracking	Sept 17	10–12	
3	Single target tracking	Sept 29	13–15	Ex 1
4	Multi-target tracking (1/2): GNN, JPDA	Oct 6?	13-15	Ex 2
5	Multi-target tracking (2/2): MHT	Fall		Ex 3
6	Random Finite Sets: PHD, etc	Fall		
7	Guest lecture	Fall?		
8	Various topics (TkBD, T2T, ETT)	Fall		

- Lectures are in Systemet, unless otherwise stated, and via Zoom.
  (Details has been mailed out!)
- Exercises are due at the end of the course.
  (Doing them as the course progresses is highly recommended!)
- Dates are preliminary, check homepage and mails for updates.

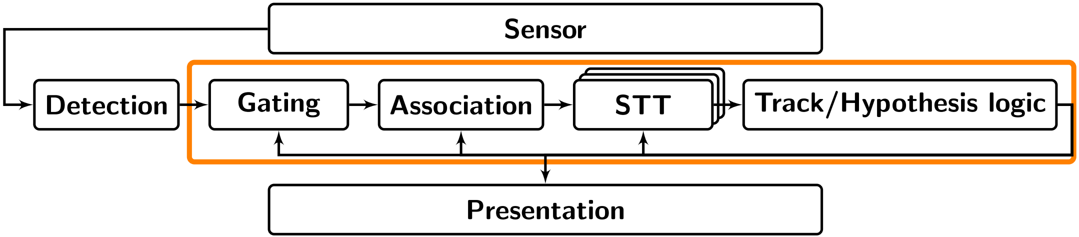
Course Literature

- Selected papers handed out during the course will be enough to follow the course.
- For a farily complete overview of the target tracking problem, methods, and algorithm collected in one place, the following books are good entry points.

  - S. S. Blackman and R. Popoli. *Design and analysis of modern tracking systems.* Artech House radar library. Artech House, Inc, 1999. ISBN 1-5853-006-0.
  - Y. Bar-Shalom, P. Willett, and T. Xin. *Tracking and Data Fusion: A Handbook of Algorithms.* Yaakov Bar-Shalom Publishing, 2011. ISBN 0-9648-3-127-9.

Multi-Target Tracking Overview

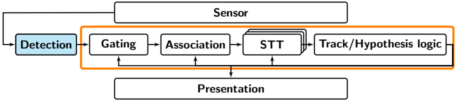
Multi-Target Tracking: conceptual view



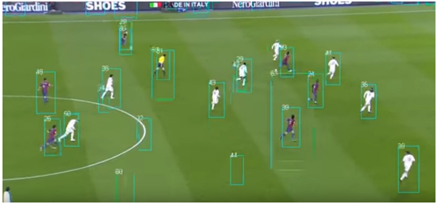
Components

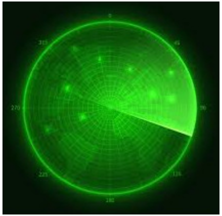
1. Detections/Observations
2. Gating
3. Association
4. Single-target tracking
5. Track and hypothesis logics
6. Presentation

Multi-Target Tracking: detection

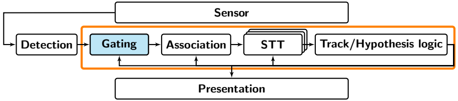


- Considered done in this course
- Sensor level signal processing
- Heavily sensor dependent

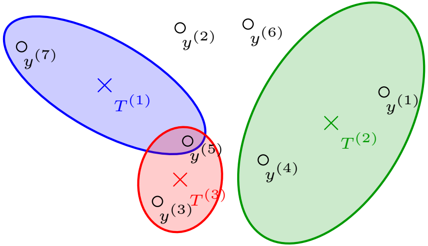




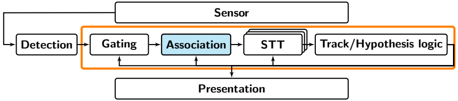
Multi-Target Tracking: gating

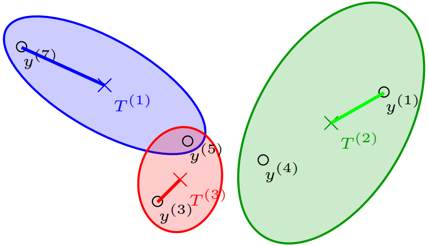


- Determine which observations could come from known targets
- Reduce tracking complexity



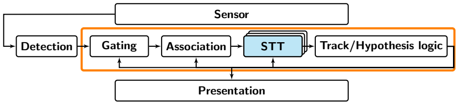
Multi-Target Tracking: association



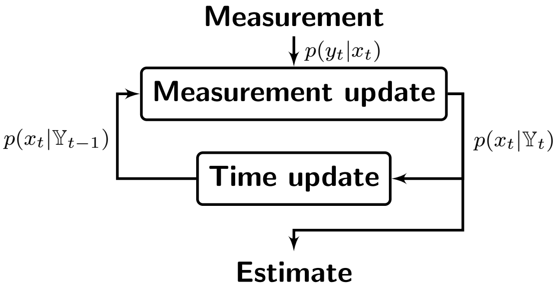


- Match observations to targets
- One or many different associations

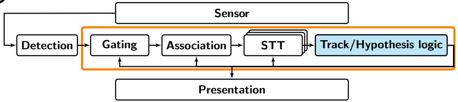
Multi-Target Tracking: STT



- Performed for each target independently, given associated observations
- Standard methods: EKF, UKF, PF, ...
- Yields state and uncertainty, given the association hypothesis

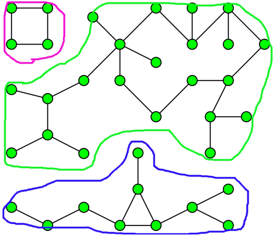


Multi-Target Tracking: track/hypothesis logic



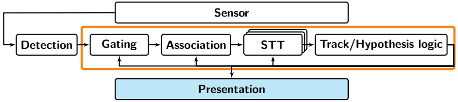
- Compute probability of given track/association hypothesis
- Track management: birth, death
- Clustering for efficiency

Track logic



Clustering independent parts

Multi-Target Tracking: presentation



- How to present the result?
- Not addressed in the course

Tracking Examples

Selected examples

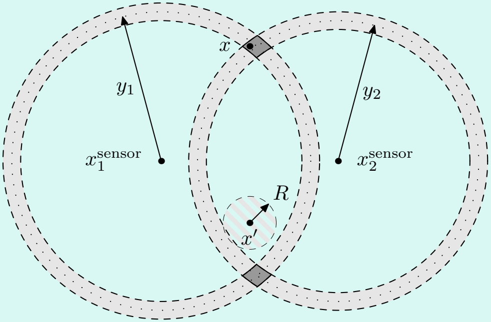
Selected examples (single target tracking/filtering and multiple target tracking):

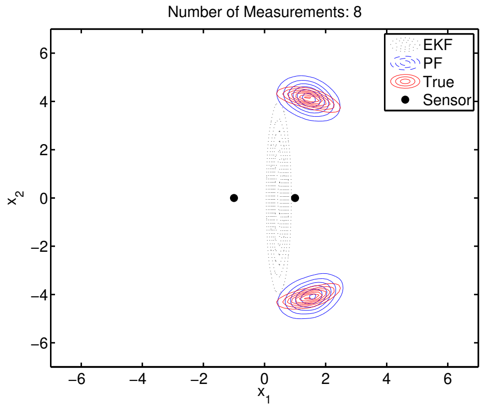
- **STT** Range-only measurements
- **STT** Positioning based on a tracking sensor
- **STT** Multiple models for maneuvering target tracking (IMM)
- **STT** Track before detect
- **MTT** Nearest Neighbor CV-model
- **MTT** MHT
- **MTT** PHD-filtering

STT: Range-Only Tracking

Range-Only Measurements

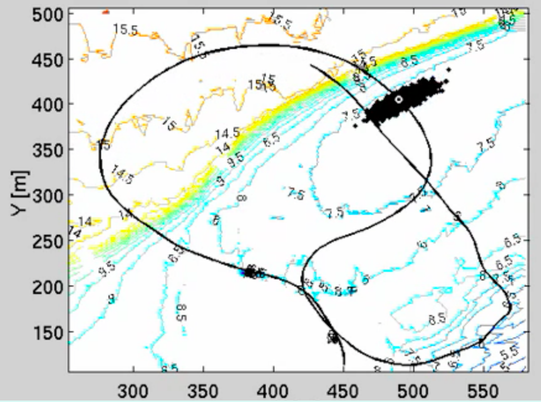
Performance, and performance measures for RO:





STT: UW map-aided navigation

UW navigation



http://youtu.be/JxUjVEn87yE

- Underwater vessel measures its own depth and distance to bottom, and sea chart provides depth $h(x_t)$.
- Video shows how a uniform prior quickly converges to a unimodal particle cloud. Note how the cloud changes form when passing the ridge.

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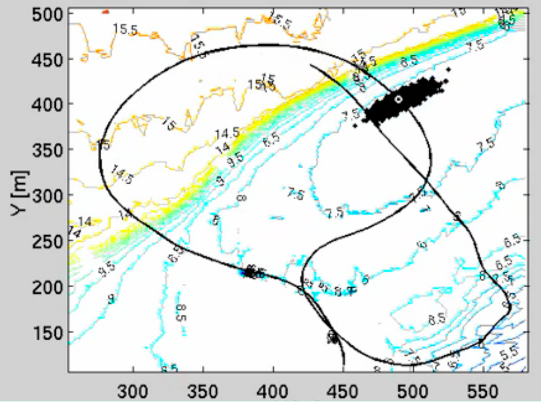

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- Underwater vessel measures its own depth and distance to bottom, and sea chart provides depth $h(x_t)$.
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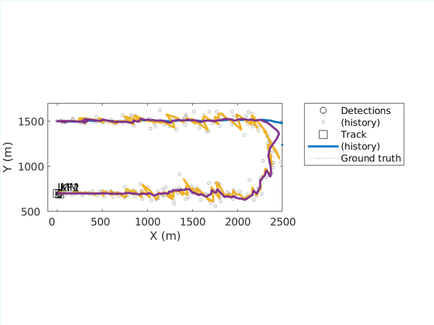
STT: Maneuvering Target

The IMM method for two models

A radar tracking application is presented using the IMM method with two filters. One filter is used to handle a straight flying path accurately, whereas the other is used to manage maneuvers. Due to the nonlinearities in the measurement equation an EKF is used for the estimation.



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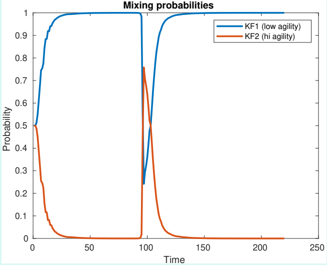

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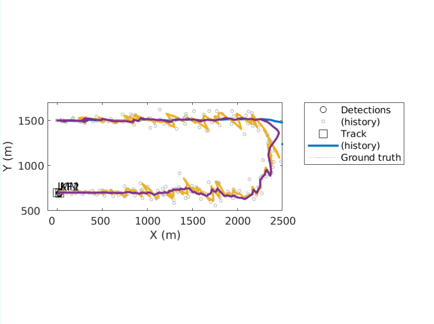

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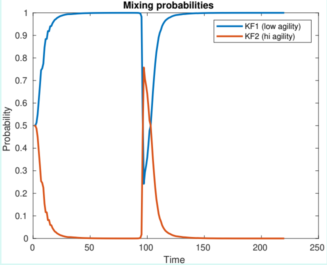
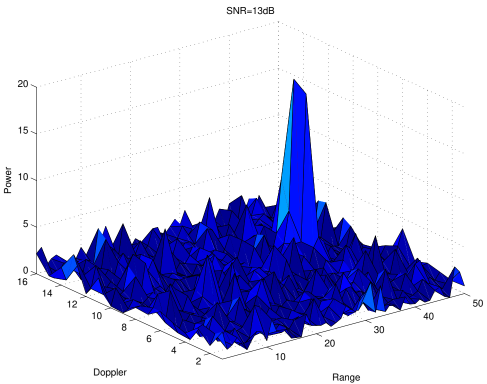


STT: Track-Before-Detect (TkBD)

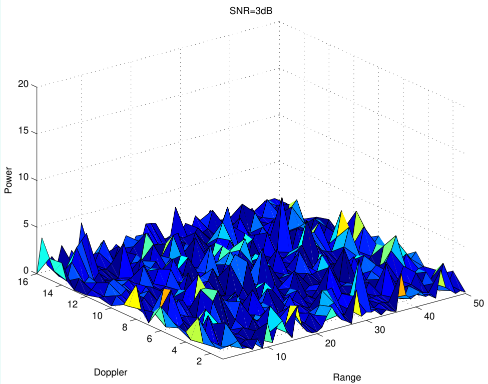
Track without first detecting the target

SNR=13 dB



Easy to detect a point target.

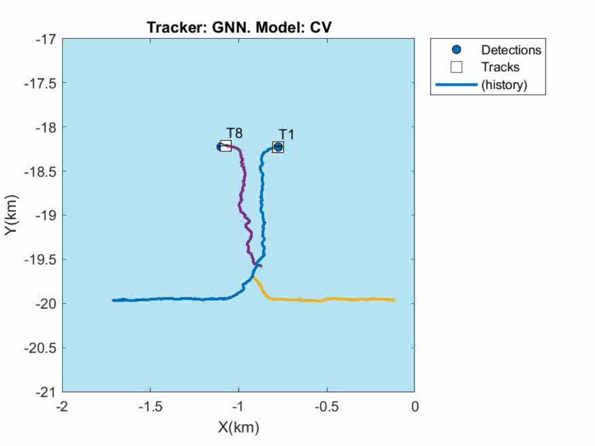
SNR=3 dB



Hard to detect a point target.

MTT: GNN CV-model

Global nearest neighbor tracking

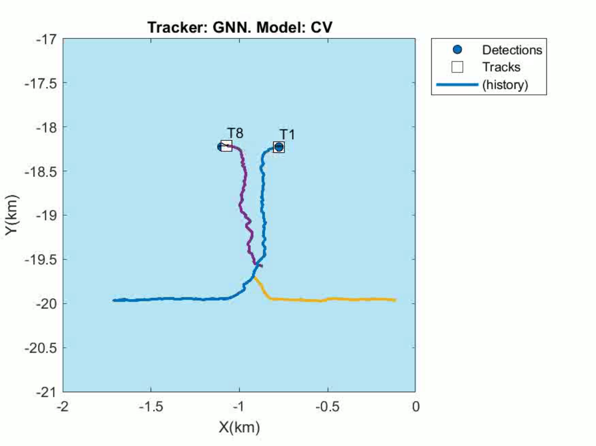


https://youtu.be/WPA2z-kw1wg

- *Global nearest neighbor* (GNN) tracker
- Simple *constant velocity* (CV) model
- Problems handling the mixed level of agility

MTT: GNN CV-model

Global nearest neighbor tracking



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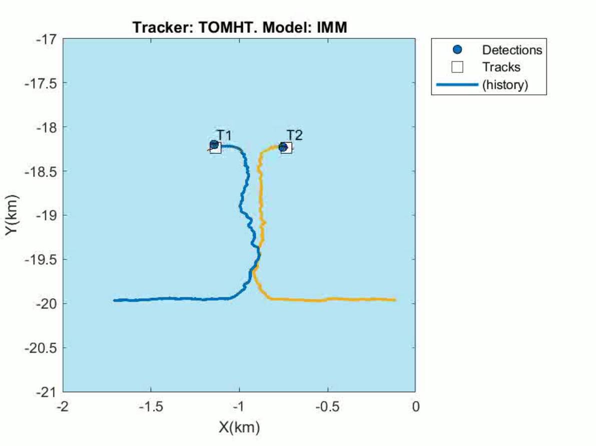
MTT: MHT IMM

Multi-hypothesis tracking

- *Multi-hypothesis tracker* (MHT) resolves measurement ambiguities
- *Interacting multiple models* (IMM) better captures the mixed level of agility



https://youtu.be/ke37FrXg_1M

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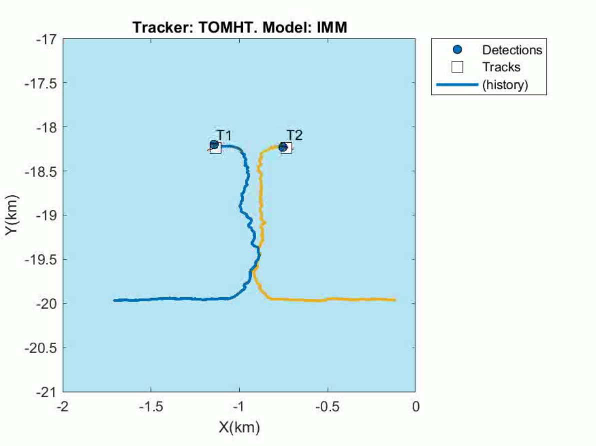

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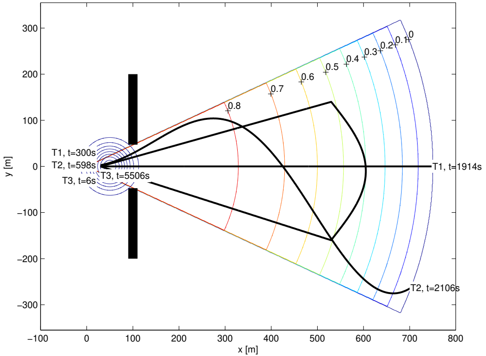
MTT: PHD Filter Example

Random finite set tracking

Standard GMPHD filter



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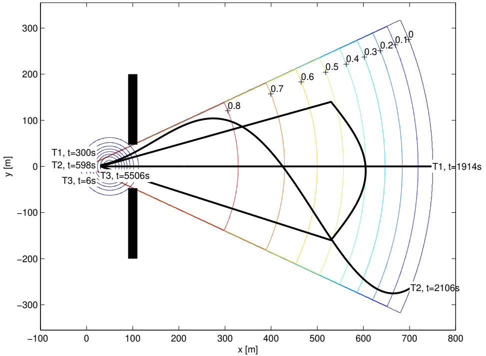
Modified GMPHD filter



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- Probability of detection dies off as a 3rd-degree polynomial, inspired by real data

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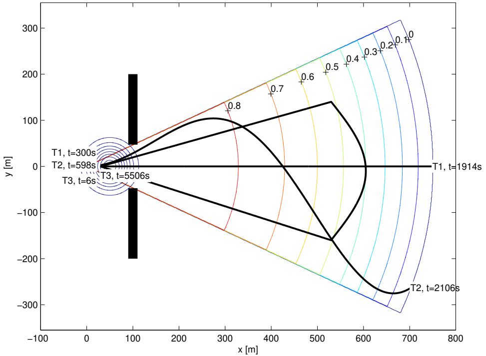

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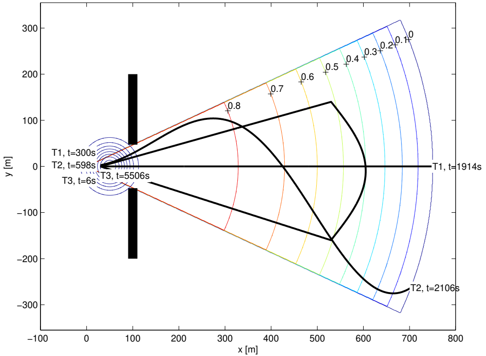

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Tracking Preliminaries

Introduction to Target Tracking (TT)

Definition: Target

A **target** is anything whose state (x) is of interest to us.

- The state can change over time with a dynamics which is itself unknown.
- Measurements/detections/observations (y^i) comes from uncertain origin.
- There are false measurements, $P_{\text{FA}} > 0$.
- Some measurements are missing, $P_{\text{D}} < 1$.
- Generally have no initial guess or estimate of the target state.

Definition: Target tracking

Target tracking is the estimation of the number of targets present in the tracking volume and theirs states.
In its most general and abstract form, it is a special case of dynamic estimation theory.

Targets and Tracks

Definition: Track

A **track** is a sequence of measurements that has been decided or hypothesized by the tracker to come from a single source.

- Usually, instead of the list of actual measurements, sufficient statistics is maintained, *e.g.*, mean and covariance in the case of a KF, particles in the case of a PF.
- In general, each measurement must be classified as either belonging to an existing track, a new track, or as being a false measurement.

Target Types

Point target A target that can result in at most a single measurement in a scan.

- This means its extension is comparable to the sensor resolution.
- However, an extended target can also be treated as a point target by tracking its centroid or corners.

Extended target A target that can result in multiple measurements in a single scan.

Unresolved targets This denotes a group of close targets that can collectively result in measurements in the sensor.

Dim target This is a target whose signal energy is very low. These can be tracked much better with *track before detect* (TkBD) type approaches.

Bayesian Problem Formulation and Solution

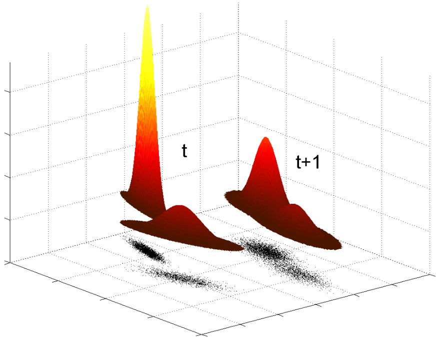
- The state x_t of interest
- Given measurements/observations $\mathbb{Y}_t = \{y_1, \dots, y_t\}$
- System model:

$$\begin{aligned} x_t &= f(x_{t-1}, w_{t-1}) && \longleftrightarrow && p(x_t|x_{t-1}) \\ y_t &= h(x_t) + e_t && \longleftrightarrow && p(y_t|x_t) \end{aligned}$$

  where w_{t-1} and e_t are stochastic processes

- Bayesian solution

$$p(x_t|\mathbb{Y}_t) = \int \frac{p(y_t|x_t)p(x_t|x_{t-1})p(x_{t-1}|\mathbb{Y}_{t-1})}{p(y_t|\mathbb{Y}_{t-1})}\, dx_{t-1}$$



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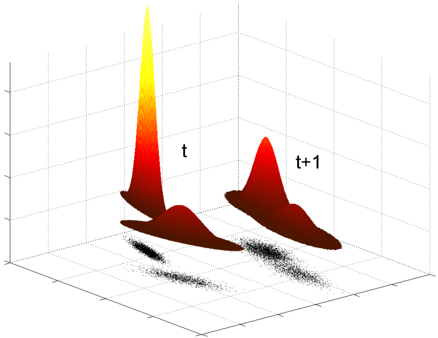

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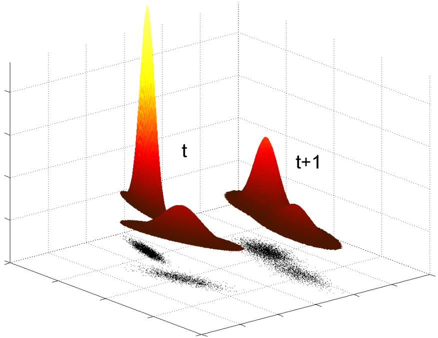

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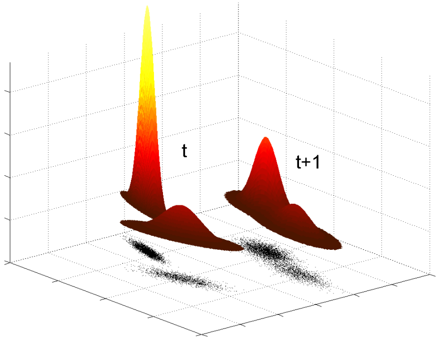

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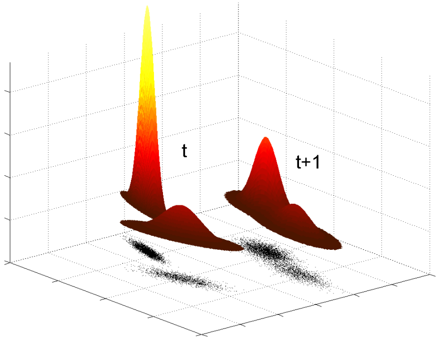


Bayesian Framework for Estimation

- Bayesian solution

$$p(x_t|\mathbb{Y}_{t-1}) = \int p(x_t|x_{t-1})p(x_{t-1}|\mathbb{Y}_{t-1})\,dx_{t-1} \tag{TU}$$

$$p(x_t|\mathbb{Y}_t) = \frac{p(y_t|x_t)p(x_t|\mathbb{Y}_{t-1})}{p(y_t|\mathbb{Y}_{t-1})} \tag{MU}$$

- Two stage procedure:
  - Time update (TU): Predict the future
  - Measurement update (MU): Correct prediction based on measurement
- Only a few analytic solutions:
  - Linear Gaussian model ⇒ Kalman filter (KF)
  - Hidden Markov model (HMM)

Bayesian Framework for Estimation

- Bayesian solution

$$p(x_t|\mathbb{Y}_{t-1}) = \int p(x_t|x_{t-1})p(x_{t-1}|\mathbb{Y}_{t-1})\,dx_{t-1} \qquad \text{(TU)}$$

$$p(x_t|\mathbb{Y}_t) = \frac{p(y_t|x_t)p(x_t|\mathbb{Y}_{t-1})}{p(y_t|\mathbb{Y}_{t-1})} \qquad \text{(MU)}$$

- Two stage procedure:
  - Time update (TU): Predict the future
  - Measurement update (MU): Correct prediction based on measurement
- Only a few analytic solutions:
  - Linear Gaussian model $\Rightarrow$ Kalman filter (KF)
  - Hidden Markov model (HMM)
- In most cases approximations are needed:
  - Analytic
  - Stochastic

Filtering

Common filters used for tracking:

- *Kalman filter* (KF)
- *Extended Kalman filter* (EKF)
- *Unscented Kalman filter* (UKF)
- *Particle filter* (PF)
- Filter banks, *e.g.*, *interacting multiple models* (IMM)

We will assume basic knowledge of first ones and only give a brief introduction here. Next lecture will deal with models used in tracking, and filter banks.

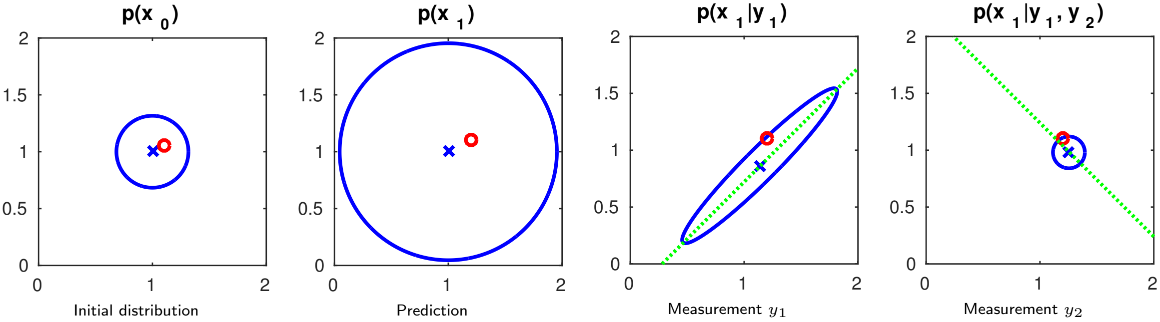
Kalman Filter (KF)

- Probably the most used filter in practice.
- Applies to linear state-space models:

$$
\begin{aligned}
x_{t+1} &= F_t x_t + G_t w_t, & \text{cov}(w_t) &= Q_t \\
y_t &= H_t x_t + e_t, & \text{cov}(e_t) &= R_t
\end{aligned}
$$

- Shown to be optimal if the noise is Gaussian, otherwise the best linear unbiased estimator (BLUE).
- Can be implemented efficiently.

Kalman Filter: illustration



Extended Kalman Filter (EKF)

Standard Algorithm

- **Initialization:** $\hat{x}_{0|0} = x_0$ and $P_{0|0} = \Pi_0$.
- **Time update:**

$$\hat{x}_{t|t-1} = f(\hat{x}_{t-1|t-1})$$
$$P_{t|t-1} = F_{t-1} P_{t-1|t-1} F_{t-1}^T + G_{t-1} Q_{t-1} G_{t-1}^T$$

- **Measurement update:**

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + K_t \big(y_t - h(\hat{x}_{t|t-1})\big)$$
$$P_{t|t} = P_{t|t-1} - K_t H_t P_{t|t-1},$$

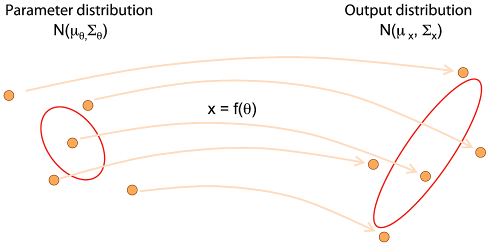
where

$$K_t = P_{t|t-1} H_t^T \left(H_t P_{t|t-1} H_t^T + R_t\right)^{-1}$$
$$f_t^T = \nabla_x f^T(x)\big|_{x=\hat{x}_{t|t}}, \qquad H_t^T = \nabla_x h^T(x)\big|_{x=\hat{x}_{t|t-1}}$$

Unscented Kalman Filter (UKF)

Fundamental idea:

Use the unscented transform (UT) to transform stochastic variables when needed.



Generate $2n_x + 1$ *sigma points*, transform these, and fit a Gaussian distribution:

$$x^{(0)} = \hat{x}$$

$$x^{(\pm i)} = \hat{x} \pm \sqrt{n_x + \lambda} P^{1/2}_{:,i}, \qquad i = 1, 2, \ldots, n_x$$

$$z^{(i)} = g(x^{(i)})$$

$$\mathsf{E}(z) \approx \sum_{i=-n_x}^{n_x} \omega^{(i)} z^{(i)} \qquad \mathsf{cov}(z) \approx \sum_{i=-n_x}^{n_x} \omega_c^{(i)} \big(z^{(i)} - \mathsf{E}(z)\big)\big(z^{(i)} - \mathsf{E}(z)\big)^T$$

Unscented Kalman Filter Algorithm (1/2)

Algorithm: time update

$$\hat{x}_{t|t-1} = \sum_{i=0}^{N} \omega_t^{(i)} x_{t|t-1}^{(i)}$$

$$P_{t+1|t} = \sum_{i=0}^{N} \omega_{c,t}^{(i)} \big(x_{t|t-1}^{(i)} - \hat{x}_{t|t-1}\big)\big(x_{t|t-1}^{(i)} - \hat{x}_{t|t-1}\big)^T$$

$$x_{t|t-1}^{(i)} = f(x_{t-1|t-1}^{(i)}, w_t^{(i)})$$

$$\omega^{(0)} = \frac{\lambda}{n_x + \lambda} \qquad \omega_c^{(0)} = \omega^{(0)} + (1 - \alpha^2 + \beta)$$

$$\omega^{(\pm i)} = \frac{1}{2(n_x + \lambda)} \qquad \omega_c^{(\pm i)} = \omega^{(\pm i)}$$

Unscented Kalman Filter Algorithm (2/2)

Algorithm: measurement update

$$\hat{x}_{t|t} = \hat{x}_{t|t-1} + P^{xy}_{t|t-1} P^{-yy}_{t|t-1} (y_t - \hat{y}_t)$$

$$P_{t|t} = P_{t|t-1} - P^{xy}_{t|t-1} P^{-yy}_{t|t-1} P^{xyT}_{t|t-1}$$

$$y_t^{(i)} = h(x^{(i)}_{t|t-1}, e_t^{(i)})$$

$$\hat{y}_t = \sum_{i=0}^{N} \omega_t^{(i)} y_t^{(i)}$$

$$P^{yy}_{t|t-1} = \sum_{i=0}^{N} \omega_{c,t}^{(i)} \big(y_t^{(i)} - \hat{y}_t\big)\big(y_t^{(i)} - \hat{y}_t\big)^T$$

$$P^{xy}_{t|t-1} = \sum_{i=0}^{N} \omega_{c,t}^{(i)} \big(x^{(i)}_{t|t-1} - \hat{x}_{t|t-1}\big)\big(y_t^{(i)} - \hat{y}_t\big)^T.$$

Unscented Kalman Filter: design parameters

- λ is defined by $\lambda = \alpha^2(n_x + \kappa) - n_x$.
- α controls the spread of the sigma points and is suggested to be chosen around 10^{-3}.
- β compensates for the distribution, and should be chosen to $\beta = 2$ for Gaussian distributions.
- κ is usually chosen to zero.

Note

- $n_x + \lambda = \alpha^2 n_x$ when $\kappa = 0$.
- The weights sum to one for the mean, but sum to $2 - \alpha^2 + \beta \approx 4$ for the covariance. Note also that the weights are not necessarily in $[0, 1]$.
- The mean has a large negative weight!

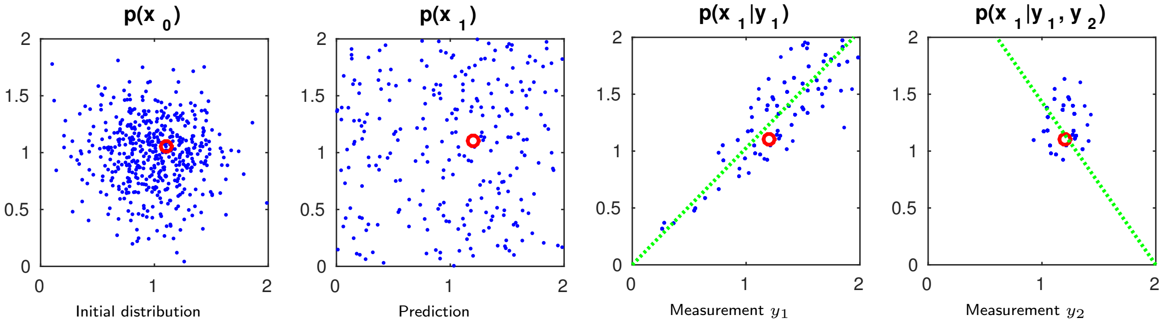
Particle Filter (PF)

Postulate a discrete approximation of the posterior. For the predictive density, we have

$$\hat{p}(x_t|\mathbb{Y}_t) = \sum_{i=1}^{N} w_{t|t-1}^{(i)} \delta(x_t - x_t^{(i)}).$$

Simulate each particle (sample) independently, and compare how well they match the obtained measurements. Use the law of large numbers.

Particle Filter: illustration



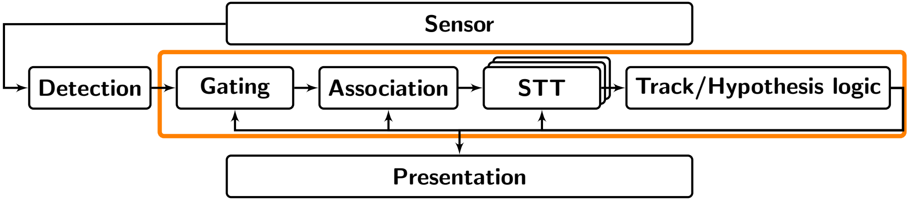
Particle Filter: algorithm

Sampling Importance Resampling (SIR) Algorithm

- **Initialize:** Generate N samples $\{x_{0|0}^{(i)}\}_{i=1}^{N}$ from $p_{x_0}(x_0)$.
- **Time update:** Simulate new particles, *i.e.* $x_{t|t-1}^{(i)} = f(x_{t-1|t-1}^{(i)}, w_{t-1}^{(i)})$, $i = 1, \dots, N$, where $w_{t-1}^{(i)} \sim p_w(w_{t-1})$,
- **Measurement update:** Compute the weights $\omega_t^{(i)} \propto p(y_t | x_{t|t-1}^{(i)})$ and normalize so they sum to one, $\sum_i \omega_t^{(i)} = 1$.
- **Resample:** Generate a new set $\{x_{t|t}^{(i)}\}_{i=1}^{N}$ by resampling with replacement N times from $\{x_{t|t-1}^{(j)}\}_{j=1}^{N}$, where $\Pr(x_{t|t}^{(i)} = x_{t|t-1}^{(j)}) = \omega_t^{(j)}$.

Summary

Summary



- Multi-target tracking is the problem of decide how many targets are present and how they move, given measurements with imperfections.
- Classic MTT can be divided in several stages: gating, association, single target tracking, track/hypothesis logic, and presentation.
- Single target tracking: Kalman type filters, particle filters

Decide what your ambitions are for the course!

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